Physics 494 HRW — 93, 94, 104, 105, 106 SHM

NAME: KEY

93 An engineer has an odd-shaped 10 kg object and needs [illegible]nd its rotational inertia about an axis through its center of [illegible]s. The object is supported on a wire stretched along the [illegible]ed axis. The wire has a torsion constant $\kappa = 0.50\ N\cdot m$. If [illegible] torsion pendulum oscillates through 20 cycles in 50 s, [illegible] is the rotational inertia of the object?

94 A grandfather clock has a [illegible]ulum that consists of a thin [illegible] disk of radius $r = 15.00$ cm [illegible] mass 1.000 kg that is attached [illegible] long thin rod of negligible [illegible]. The pendulum swings freely [illegible] an axis perpendicular to [illegible] rod and through the end of [illegible] rod opposite the disk, as [illegible] in Fig. 15-54. If the pendu[illegible] to have a period of 2.000 s [illegible] small oscillations at a place [illegible] $g = 9.800\ m/s^2$, what must [illegible] rod length L to the nearest [illegible] of a millimeter?

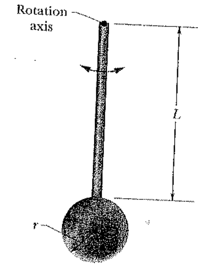


FIG. 15-54 Problem 94.

104 A simple harmonic oscillator consists of a block attached to a spring with $k = 200$ N/m. The block slides on a frictionless surface, with equilibrium point $x = 0$ and amplitude 0.20 m. A graph of the block's velocity v as a function of time t is shown in Fig. 15-58. The horizontal scale is set by $t_s = 0.20$ s. What are (a) the period of the SHM, (b) the block's mass, (c) its displacement at $t = 0$, (d) its acceleration at $t = 0.10$ s, and (e) its maximum kinetic energy?

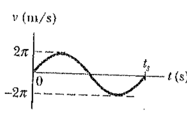


FIG. 15-58 Problem 104.

105 A simple harmonic oscillator consists of a 0.50 kg block attached to a spring. The block slides back and forth along a straight line on a frictionless surface with equilibrium point $x = 0$. At $t = 0$ the block is at $x = 0$ and moving in the positive x direction. A graph of the magnitude of the net force $\vec{F}$ on the block as a function of its position is shown in Fig. 15-59. The vertical scale is set by $F_s = 75.0$ N. What are (a) the amplitude and (b) the period of the motion, (c) the magnitude of the maximum acceleration, and (d) the maximum kinetic energy?

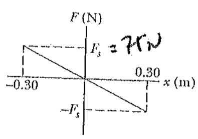


FIG. 15-59 Problem 105.

106 In Fig. 15-60, a solid cylinder attached to a horizontal spring ($k = 3.00$ N/m) rolls without slipping along a horizontal surface. If the system is released from rest when the spring is stretched by 0.250 m, find (a) the translational kinetic energy and (b) the rotational kinetic energy of the cylinder as it passes through the equilibrium position. (c) Show that under these conditions the cylinder's center of mass executes simple harmonic motion with period

$$T = 2\pi\sqrt{\frac{3M}{2k}},$$

where M is the cylinder mass. (*Hint:* Find the time derivative of the total mechanical energy.)

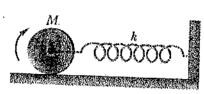
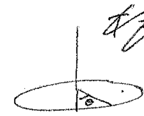


FIG. 15-60 Problem 106.

(93) $T = \frac{50\,s}{20} = 2.5\,s$

unit check: $\frac{N\cdot m}{kg\cdot m^2} = \frac{kg\,m}{s^2\,kg\,m} = \frac{1}{s^2}$



$\Sigma\tau = I\alpha = -\kappa\theta$

$\alpha = -\frac{\kappa}{I}\theta \qquad \omega = \sqrt{\frac{\kappa}{I}} = \frac{2\pi}{T}$

$\boxed{I = 0.079\ kg\cdot m^2}$ $\qquad T = 2\pi\sqrt{\frac{I}{\kappa}} \rightarrow I = \frac{T^2\kappa}{4\pi^2}$

(94) For physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgh_{cm}}}$ $\quad h$ = dist. from axis to c.m.

$I = I_{cm} + md^2 = \frac{1}{2}mr^2 + m(L+r)^2$

$\frac{T^2(mg(L+r))}{4\pi^2} = \frac{1}{2}mr^2 + m(L+r)^2$

$T^2(mg(L+r)) = 2\pi^2 mr^2 + m(L+r)^2$

solve for L ...

(104) (a) 0.2 s

(b) $T = 2\pi\sqrt{\frac{m}{k}} \rightarrow m = \frac{T^2k}{4\pi^2} = 0.2\ kg$

(c) $v_{max} = 2\pi = \omega A \qquad \omega = \frac{2\pi}{T} = 31.4\ rad/s$

$A = 0.2\ m$

(d) $a_{max} = \omega^2 A = 197\ m/s^2$

(e) $\frac{1}{2}mv_{max}^2 = 3.95\ J$

(105) (a) $A = 0.3\ m$

(b) $T = 2\pi\sqrt{m/k} \qquad k = \text{slope} = 75/0.3 = 250\ N/m$

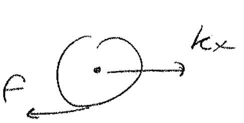
$\boxed{T = 0.281\ s}$

(c) $a_{max} = F/m = 75/0.5 = \boxed{150\ m/s^2}$

(d) $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2 = \frac{1}{2}(250)(0.3)^2 = 11.3\ J$

(106) $\frac{1}{2}kA^2 = KE_{rot} + KE_{trans} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

(c)



$kx - f = ma \qquad fr = I\alpha = \frac{1}{2}mr^2\alpha$

$f = \frac{1}{2}mr\alpha = \frac{\frac{1}{2}mra}{r} = \frac{1}{2}ma$

$kx - \frac{1}{2}ma = ma$

$1.5ma = kx$

$a = \underbrace{\left(\frac{k}{1.5m}\right)}_{\omega^2}x \rightarrow \omega = \sqrt{\frac{2k}{3M}} \rightarrow T = 2\pi\sqrt{\frac{3M}{2k}}$