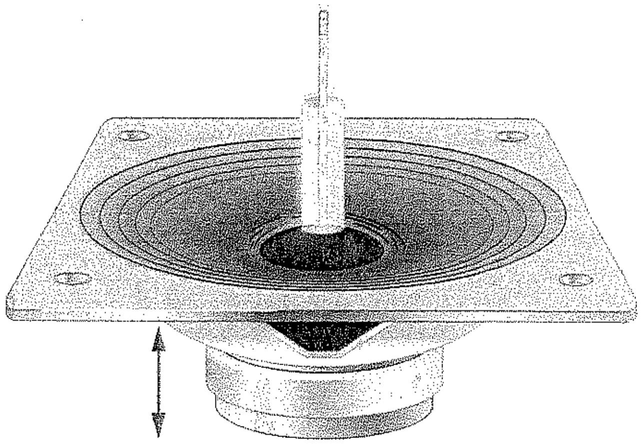
AP Physics

SHM Practice B

Name: KEY

13.81 (modified) A vertical, hollow tube is connected to a speaker, which vibrates vertically with SHM as shown in the figure to the right. The speaker operates with constant amplitude A, but variable frequency, f. A slender pencil of mass m is placed inside the tube. At low frequencies the pencil stays in contact with the speaker at all times. At higher frequencies the pencil begins to rattle.

(the Lamoreax HS contest question he will never forget)



a. Using the dot below, draw the FBD and label the forces acting on the pencil at any time when it is in contact with the speaker.

(2)

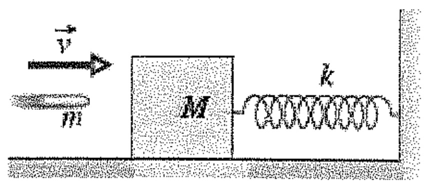
b. What must be the minimum acceleration of the speaker if the pencil is to barely lose contact?

$9.8\ m/s^2$ ($10\ m/s^2$) downward (1)

c. Find an equation for the frequency at which rattling begins.

$$a_{max} = \omega^2 A \longrightarrow \omega = \sqrt{\frac{a}{A}} = \sqrt{\frac{g}{A}}$$ (2)

2. A block of mass *M*, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant *k*. A bullet of mass *m* and velocity *v* strikes the block as shown to the right. The bullet is embedded in the block. Determine
(a) the speed of the block immediately after the collision, and
(b) the amplitude of the resulting simple harmonic motion.



(a) ~~cons~~ momentum conservation

$$mv = (m+M)v' \qquad v' = \boxed{\frac{m}{m+M}(v)}$$ (2)

(b) cons. of energy ~~KE~~

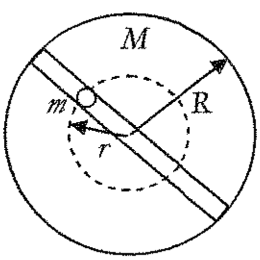
$$\frac{1}{2}(m+M)(v')^2 = \frac{1}{2}kA^2$$

$$A = \sqrt{\frac{(m+M)(v')^2}{k}} = \sqrt{\frac{(m+M)}{k}}\left(\frac{m}{m+M}\right)v = \boxed{\frac{mv}{\sqrt{k(m+M)}}}$$ (2)

Physics 492

TUNNEL THROUGH THE EARTH – AN EXAMPLE

Name: WEM



A tunnel is drilled through the center of a planet of mass M and radius R. The planet has uniform density.

1. Determine the amount of mass M' (in terms of M) that is contained in the smaller sphere of radius r.

$$\frac{M'}{M} = \frac{r^3}{R^3}$$ (see class notes from gravitation)

$$M' (M)(r^3/R^3)$$

2. Determine the gravitational force acting on an object of mass m that is released at the entrance of the tunnel. Answer should be in terms of G, m, M', and r.

$$F_g = \frac{GmM'}{r^2}$$

3. Determine the acceleration of the object. Answer should be in terms of G, m, M', and r.

$$\cancel{m}a = \frac{G\cancel{m}M'}{r^2} \rightarrow a = \frac{GM'}{r^2}$$

4. Substitute your answer for M' from #1 above into your answer for #3 to get a final value for the acceleration.

$$a = \frac{G}{(r^2)}\left(\frac{Mr^3}{R^3}\right) = \left(\frac{GM}{R^3}\right)r$$

5. Recall that $a = -\omega^2 r$ is the hallmark of SHM. Determine the frequency f of the oscillation of the mass m as it falls back and forth through the tunnel.

$$\omega = 2\pi f = \sqrt{\frac{GM}{R^3}} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{GM}{R^3}}$$

6. Determine the frequency of a *circular* orbit with a radius R. Compare this to your result from #5 above.

$$v = \frac{2\pi R}{T} = 2\pi R f$$

$$\frac{G\cancel{m}M}{R^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{R}} \rightarrow v^2 = \frac{GM}{R} \rightarrow 4\pi^2R^2f^2 = \frac{GM}{R}$$

$$f = \sqrt{\frac{GM}{4\pi^2R^3}} = \frac{1}{2\pi}\sqrt{\frac{GM}{R^3}}$$

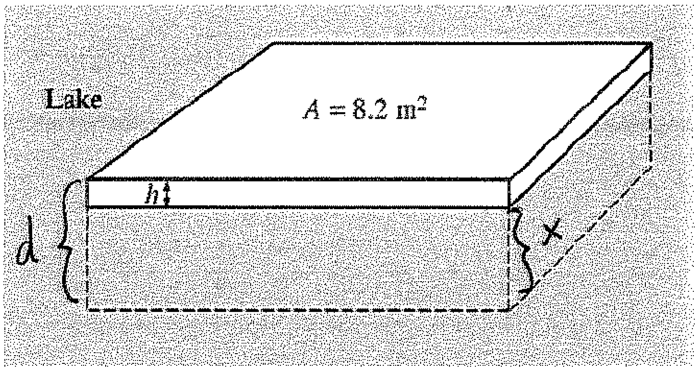
Same

AP Physics

2005#5 Modified

(Buoyancy and SHM)

Name: KEY



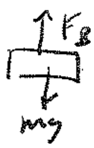
Note: Figure not drawn to scale.

A large rectangular raft ($\rho = 650\ \text{kg/m}^3$) is floating on a lake. The surface area of the top of the raft is $8.2\ \text{m}^2$ and its volume is $1.80\ \text{m}^3$. The density of the lake water is $1000\ \text{kg/m}^3$.

a. Calculate the height h of the portion of the raft that is above the surrounding water.

First find d (see picture) $\rightarrow V = Ad \rightarrow d = V/A = 1.8/8.2 = 0.22\,m$

Raft in equilibrium so



$F_B = mg \Rightarrow \rho_{water} V g = \rho_{raft} V_{raft}\, g \rightarrow V_{water} = \frac{\rho_{raft} V_{raft}}{\rho_{water}} = \frac{650(1.8)}{1000}$

$V_{water} = 1.17\,m^3 = Ax \rightarrow x = 0.14\,m$ so $h = 0.22 - 0.14 = \boxed{0.08\,m}$

b. Calculate the magnitude of the buoyant force on the raft and state its direction.

equal to weight of raft

$= \rho_{raft} V_{raft}\, g = 650(1.8)(10) = \boxed{11700\,N}$

c. If the average mass of a person is 75 kg, calculate the maximum number of people that can be on the raft without the top of the raft sinking below the surface of the water. (Assume that the people are evenly distributed on the raft.)

part ~~of~~ out of water $\rightarrow V_{out} = 8.2(0.08) = 0.656\,m^3$ 1 person ≈ 750N

$F_B = \rho_{water} V_{out}\, g = 1000(0.656)(10) = 6560\,N$ $\frac{6560}{750} = \boxed{8 \text{ persons}}$

d. With zero people on the raft, a large barrel is placed on it and it sinks a distance of $h/4$ into the water. The ~~person~~ barrel then ~~jumps~~ rolls off of the raft and ~~it~~ the raft oscillates up-and-down in simple harmonic motion.

i. What is the amplitude of the SHM?

$h/4 = 0.08/4 = \boxed{0.02\,m}$

ii. What is the frequency of the SHM?

max force occurs when $h/4$ submerged $\rightarrow F_B = \rho(8.2)(\frac{h}{4})g$

$(1000)(8.2)(0.02)(10) = 1640\,N = ma_{max} = (\omega^2 A)m$ ← mass of raft

$\omega^2(h/4) = \frac{1640}{1170} \Rightarrow \omega = 8.37 = 2\pi f$

$f = \boxed{1.33\,Hz}$